Modeling in asymmetric information markets

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Outline

1. Asymmetry of information
2. Modeling of the "lemons" market "lemons"
3. Spence model
4. Asymmetry in labor relations

Asymmetry of information

Incomplete information about the object of the transaction

- obtaining information related to resource costs;
- information is not always reliable;
- economic agents are unable to remember and process all the information available to them;
- not all economic agents have sufficient knowledge and skills that would allow them to adequately process information.

Definition – 1

- **Agent** – in a contractual relationship, the party with the rights of residual control delegated to it by the other party, usually the principal. As a rule, is an informed party.
- **Principal** – in a contractual relationship, the party with the rights to residual income. It is usually an uninformed party.

Definition – 2

- **Opportunism** – pursuit of personal goals using cunning. It happens in pre-contract (negative selection) and post-contract (moral hazard and extortion) periods.
- **Post-contract opportunism** is that after the the agreement, the actions of the agent do not (fully) meet the interests of the principal. This problem arises because, firstly, the principal and the agent have different interests, and secondly, there is an asymmetry of information.
- **Asymmetry of information** – the state in which one part of the participants in the market has important information, and the other part does not have such information.

Asymmetric information

- situation in which the market with asymmetric information is buying the worst, not the best types of goods:
  - the buyer is not able to clearly differentiate between sellers offering goods and services of different quality;
  - the seller cannot classify his customers according to the costs required to service them.

- behavior of the individual that deliberately increases the likelihood of possible damage in the hope that the damage will be fully covered by the insurance company:
  - one of the parties to the contract will carry out activities undesirable from the point of view of the other party, which is not able to observe the behavior of the first.

"Unobserved characteristics "

Moral risk

Moral risk

- Moral risk in the field of insurance
  - Dishonest behavior
  - Ignoring risks
  - Excessive use of services
- Moral risk in financial markets
- Moral risk in countercyclical state policy
- Moral risk in labor relations
- Moral risk in the commodity market

Struggle with moral risk: insurance companies

- Implement more careful selection of candidates, classifying clients by risk groups.
- Refusal to insure groups of high-risk clients.
- Partial compensation.

Struggle with moral risk: labor relations

- internal (available to the interested party under a specific contract, and paid by the parties to the contract):
  - payment based on the result;
  - monitoring (revealing the real contribution of the manager);
  - management reorganization or staff rotation;
  - effective wages;
- external (provided by the market or the environment where the contract is concluded):
  - threat of takeover (corporate control market);
  - creditors' control;
  - threat of bankruptcy;
  - labor market of managers (reputation effect, career growth, mutual control of managers).

Fighting moral hazard: goods market

- signaling
  - guarantee;
  - pledge;
  - company reputation;
  - quality certificates;
  - licenses.
- standardization of products.

Social Insurance

- **Social Insurance** is a compensator for imperfections (shortcomings) of the private insurance market.
- Social insurance is provided by government agencies in areas where the private insurance market is unable to meet existing needs.
- Private companies do not undertake to insure people against unemployment.

Modeling in the market of "lemons"

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Lemon market: an example

Used car	Number	Maximum price buyer	Minimum seller price
High quality	100	600	500
Low quality	100	200	150

The information is complete and symmetrical

- There will be two independent markets.
- Consumer gain:

  $$(600 - P1)\ 100 + (200 - P2)\ 100;$$

- profits of producers:

  $$(P1 - 500)\ 100 + (P2 - 150)\ 100.$$

The information is incomplete but symmetrical

- The probability that a randomly selected car is of high quality is 0.5.
- The buyer is willing to pay the maximum price for the car Pd = 600x0.5 + 200x0.5 = $ 400, and the seller is ready to sell the car at least for Ps = 500x0.5 + 150x0.5 = $ 325
- Car sales will be 200.
- The amount of winnings of buyers and sellers will not decrease.
- Some buyers suffer losses by paying for a low-quality car more than their actual willingness to pay. However, the rest of the buyers will benefit by paying significantly less for a high-quality car than they were willing to.

The information is incomplete and asymmetric

- The seller will not want to sell a high quality car for $ 400.
- Only low quality goods will remain on the market.
- Sales - 100 cars.
- Asymmetric information about quality will reduce public welfare.

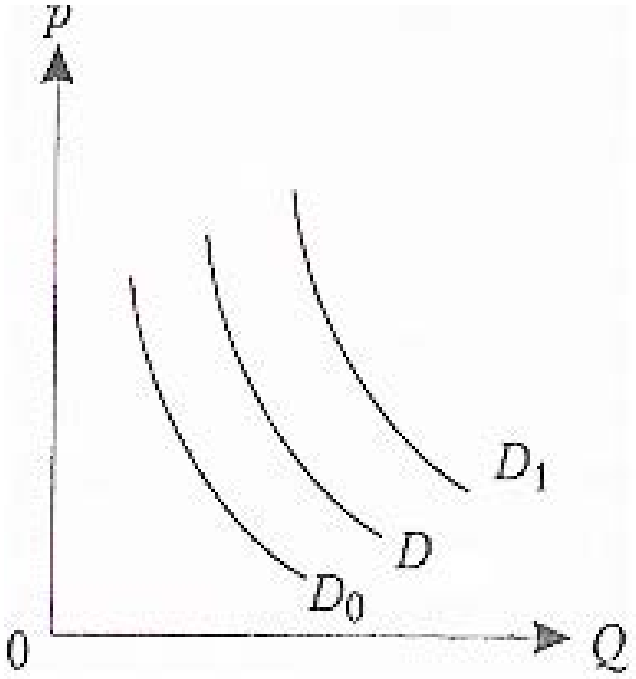
Modeling market of "lemons"

- D_0 – demand for goods of low quality;
- D_1 – demand for goods of high quality;
- S_0 – supply for low quality goods;
- S_1 – supply for high quality goods;
- a – market share of quality goods.

The volume of market demand

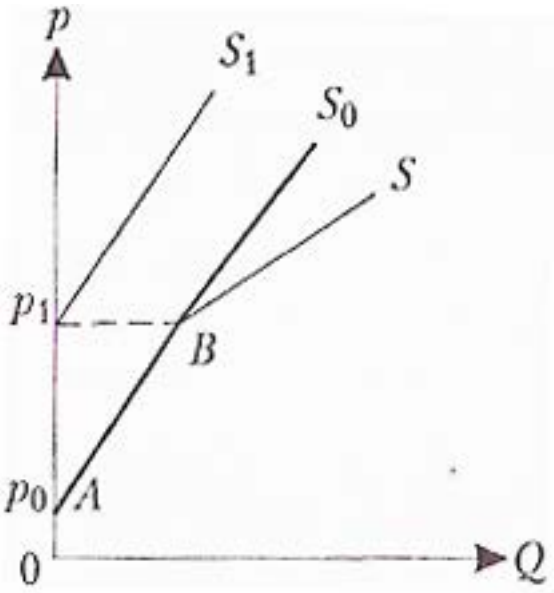
- The volume of market demand is equal to the weighted average demand for goods of different quality

$$D = aD_1 + (1-a)D_0$$



Market supply

- The minimum bid price of the seller of quality goods is indicated by p_1, and the seller of low quality – by p_0.



Fraction of quality goods

$$a = \begin{cases} 0, p \le p1, \\ S_1 / (S_1 + S_0), p > p1 \end{cases}$$

- Due to the asymmetry of market information, sellers of quality goods lose, and sellers of substandard goods win.

Static equilibrium: condition

- Demand for quality goods is given by the formula:

$$D_1 = 22 - p$$

- for low quality goods:

$$D_0 = 16 - p$$

- The offer of quality goods is given by the formula:

$$S_1 = p - 6$$

- for low quality goods:

$$S_0 = p - 2$$

Static equilibrium

$$S = \begin{cases} 0, p \le 2, \\ p - 2, 2 < p \le 6, \\ 2p - 8, p > 6. \end{cases}$$

At $p > 6$

$$a = (p-6)/(2p-8)$$

$$D = (46p - 164 - 2p^2)/(2p-8)$$

$$6p^2 - 78p + 228 = 0$$

$$p^1 = 4,44 \qquad p^2 = 8,56$$

at $2 < p < 6$

$$p - 2 = 16 - p$$

$$p = 9.$$

- Final equilibrium achieved at a price of 8.56. The share of quality goods is 0.31.

Dynamic equilibrium: condition – 1

- Demand for quality goods is given by the formula:

$$D_1 = 22 - p$$

- for low quality goods:

$$D_0 = 16 - p$$

- The offer of quality goods is given by the formula:

$$S_1 = p - 6$$

- for low quality goods:

$$S_0 = p - 2$$

Dynamic equilibrium: condition – 2

- buyers form a curve of market demand for the current day, taking into account the share of sales of quality goods in the previous day;
- on the first day, buyers form a curve of market demand based on the assumption that all goods sold on the market are of high quality.

Day 1

$$2p - 8 = 22 - p$$

$$p = 10$$

- At the price of 10 the offer of the seller of high quality goods is equal to 4, the offer of the seller of the low quality goods is equal to 8, the market offer is equal to 12, the share of high quality goods in the market is 0,33.

Day 2

- Buyers are based on the assumption of 33% of quality goods:

$$D = 0,33(22 - p) + 0,67(16 - p) = 18 - p$$

- On the second day the equilibrium price is 8.7.

Dynamics

Day	D	P	S_1	S_0	S	a
1	$22-p$	10	4	8	12	0.33
2	$18-p$	8.7	1.7	6.7	8.4	0.2
3	$17,2-p$	8.4	2.4	6.4	8.8	0.27
4	$17,6-p$	8.5	2.5	6.5	9.0	0.28
…	…	…	…	…	…	…
100	$17,86-p$	8.56	2.56	6.56	9.12	0.31

Dynamic equilibrium: condition – 1*

- Demand for quality goods is given by the formula:

$$D_1 = 22 - p$$

- for low quality goods:

$$D_0 = 8 - p$$

- The offer of quality goods is given by the formula:

$$S_1 = p - 6$$

- For low quality goods:

$$S_0 = p - 2$$

Dynamic equilibrium: condition – 2*

- buyers form a curve of market demand for the current day, taking into account the share of sales of quality goods in the previous day;
- on the first day, buyers form a curve of market demand based on the assumption that all goods sold on the market are high quality.

Day 1 *

$$2p - 8 = 22 - p$$

$$p = 10$$

- At the price of 10 the offer of the seller of the qualitative goods is equal to 4, the offer of the seller of the low quality goods is equal to 8, the market offer is equal to 12, the share of high quality goods in the market makes 0,33.

Day 2 *

- Buyers are based on the assumption of 33% of quality goods:

$$D = 0,33(22 - p) + 0,67(8 - p) = 12,62 - p$$

- On the second day the equilibrium price is 6.87.

Dynamics*

Day	D	P	S_1	S_0	S	a
1	$22-p$	10	4	8	12	0.33
2	$12,62-p$	6.87	0.87	4.87	5.74	0.15
3	$10,10-p$	6.05	0.05	4.05	5.10	0.01
4	$8,14-p$	5.07	0	3.07	3.07	0
5	$8-p$	5	0	3	3	0
…	…	…	…	…	…	…
100	$17,86-p$	5	0	3	3	0

Spence Model

Model assumptions – 1

- competitive labor market;
- two risk-neutral employers with the same technology, which has a constant impact on scale;
- the only factor of production is labor.

Model assumptions – 2

- There are two types of workers:
  - L with low productivity; produces good; Y_L
  - H with high productivity; produces good; Y_H
- $Y_L < Y_H$

Model assumptions – 3

- the employer knows that in the market:
  - the share of low-productivity workers labor ; μ_L
  - the fate of workers with high productivity labor ; μ_H
- wages do not depend on the amount of goods produced by the worker.

Model assumptions – 4

- The first employer, not knowing the type of worker, agrees to pay wages in in the amount of ω_1;
- the second agrees to give ω_2.
- The worker knows his type. He agrees to bigger of the proposed wages $\{\omega_1; \omega_2\}$.

Model – 1

- Employer's profit:

$$\mu_H Y_H + \mu_L Y_L - \omega_j, \quad j = \overline{1,2}$$

- Employee's profit:

$$\omega_j, \quad j = \overline{1,2}$$

Result

- Employee H has a disadvantage – he produces more, but he is not paid extra money, as it is not possible to determine the type of worker at the time of employment.

- It is necessary to signal to the employer about their abilities, for example, to inform the employer about their own education.

Signal

- **Signal** (a) – a certain action of a potential worker, which precedes the hiring process and serves as a sign for employers of high productivity of the worker.
- $c(a)$ – costs of action a.
- Employee utility function:

$$u(\omega, c(a)) = \omega - c(a)$$

Difference in qualifications

$$c_L(a) - c_H(a) \geq 0$$

The income produced by the worker depends on the signal

$$Y = Y(a)$$

moreover

$$Y_H(a) > Y_L(a), \forall a$$

So:

$$\omega_j = \omega_j(a)$$

Employee selection

$$\begin{cases} \omega_1 - c(a), \quad when \quad \omega_1 > \omega_2; \\ \omega_2 - c(a), \quad when \quad \omega_1 < \omega_2; \\ p\left(\omega_1 - c(a)\right) + (1-p)\left(\omega_2 - c(a)\right), \\ \qquad\qquad p = \left[0,1\right], \quad when \quad \omega_1 = \omega_2; \end{cases}$$

The choice of the employer

- The employer, focusing on the signal a, determines the expected rate of highly skilled workers, $\overline{\mu}_H$
- expected income:

$$\overline{Y}(a) = \overline{\mu}_L(a)Y_L(a) + \overline{\mu}_H(a)Y_H(a)$$

- expected profit:

$$\begin{cases} \overline{Y}(a) - \omega_1(a) \quad ma \quad 0, \quad when \quad \omega_1(a) > \omega_2(a) \\ 0 \quad ma \quad \overline{Y}(a) - \omega_2(a), \quad when \quad \omega_1(a) < \omega_2(a) \end{cases}$$

$$\omega_1(a) = \omega_2(a) = \overline{Y}(a) = \overline{\mu}_L(a)Y_L(a) + \overline{\mu}_H(a)Y_H(a)$$

Disadvantages of the model

- An unrealistic situation when education does not affect the production potential.
- "Diploma effect" – a worker with high productivity spends money and time on obtaining a diploma only in order to demonstrate high productivity and receive a high salary.

Question

- Costs on education: altruism, investments in human capital or requirement of economic growth?

Investments?

- study grants are available redistributive policy and lack of credit system;
- subsidizing education is necessary and useful because it can increase the productivity of labor capital and improve the social side of society

Assumption

- funding is provided through the "altruism" of parents - households;
- if parents are not altruistic about their children, they spend less money on education;
- parents, because of their finite lives, do not see any future benefits in such investments (short-term benchmark), which leads to the difference between social and private incomes;
- the state, which cares about the future education of its citizens, should reduce this difference by funding training.

How to measure altruism?

- Question A026 , World Values Survey:
- “Which answer is most in line with your ideas about parenting?
  - A: Parents are obliged to do everything possible for the sake of their children, even at the cost of their well-being
  - B: Parents have their own lives and they should not sacrifice them for the sake of their children
- Altruism =% of people who chose A.

Types of altruism

- natural – parents' care for their child;
- endogenous – generated by the analysis of parents of their costs and profits.

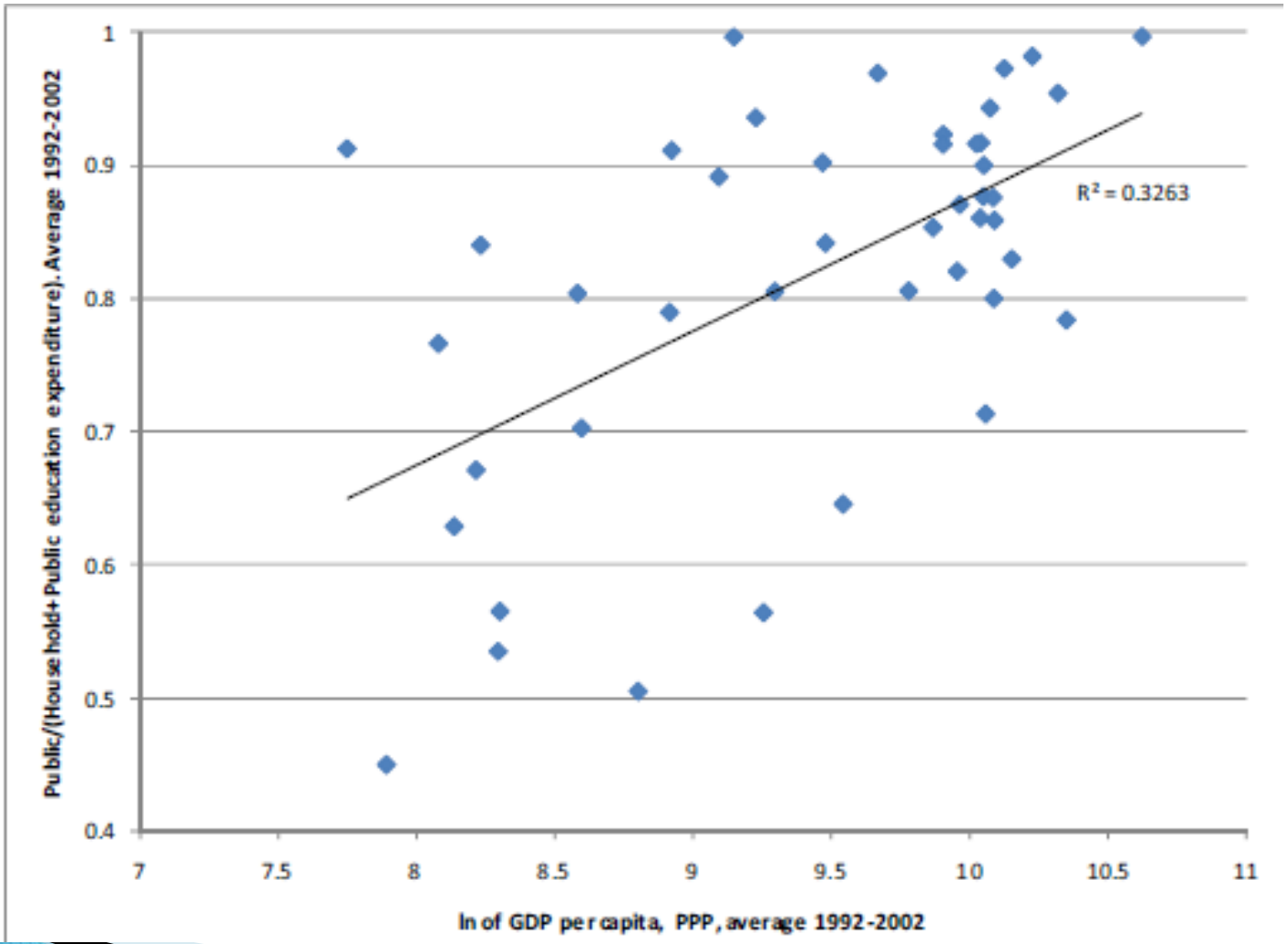
- In poorer countries, natural can prevail over endogenous, while in richer countries – vice versa.

Conclusion

- the level of public funding for education is higher in countries where parents are less prone to altruism.

- Hypothesis: Countries where parents are prone to altruism and fund their children's education are developing faster than those where state funding compensates.

Hypothesis analysis



Conclusions of the model

- Parental altruism can be an influential factor in estimating the optimal amount of education subsidies.
- In countries where parents are less altruistic about their children's education, the state should compensate by funding more education.

Asymmetry in labor relations

Moral risk in labor relations

- Moral risk is the probability that the employee after the conclusion of the contract will make insufficient efforts to perform the tasks assigned to him.

Model with two levels effort and results – 1

- The utility functions of the principal and the agent at at a certain level of effort will look like:

$$U_p = p_e(R_2 - w_2) + (1 - p_e)(R_1 - w_1)$$

$$U_A = p_e u(w_2) + (1 - p_e)u(w_1) - e$$

where

- R_1 and R_2 – two levels of results of the principal, low and high, ie $R_1 < R_2$,
- p_e – the probability of the result R_2 at a given level of effort e,
- w_1 and w_2 – levels of salary depending on the result,
- $u(w_i)$ – the utility.

Model with two levels effort and results – 2

- The probability of a high result can take the following values:

$$\begin{cases} p[e|e=e_1]=p_1 \\ p[e|e=e_2]=p_2 \end{cases}$$

where $e_1 < e_2$ *and* $0 < p_1 < p_2$.

- It is necessary to find such terms of the contract that would maximize the utility of the principal, while encouraging the agent to choose the right level of effort.

Model with two levels effort and results – 3

- The principal's task is to find conditional maximum of its utility when for agent is stimulized to choose a low level of effort, has the form:

$$U_p = p_1(R_2 - w_2) + (1 - p_1)(R_1 - w_1) \to \max_{w_1, w_2}$$

with restrictions on participation and self–selection

$$p_1 u(w_2) + (1 - p_1)u(w_1) - e_1 \geq \underline{U}$$

$$p_1 u(w_2) + (1 - p_1)u(w_1) - e_1 \geq p_2 u(w_2) + (1 - p_2)u(w_1) - e_2$$

- Wage, which provides the agent with alternative utility, is paid subject to some (low) level of effort:

$$u(w_1) - e_1 = \underline{U}$$

Model with two levels effort and results – 4

- When an agent is given an incentive to choose a high level of effort, the same problem is:

$$U_p = p_2(R_2 - w_2) + (1 - p_2)(R_1 - w_1) \to \max_{w_1, w_2}$$

- with restrictions on participation and self-selection

$$p_2 u(w_2) + (1 - p_2) u(w_1) - e_2 \geq \underline{U}$$

$$p_2 u(w_2) + (1 - p_2) u(w_1) - e_2 \geq p_1 u(w_2) + (1 - p_1) u(w_1) - e_1.$$

Example

- The utility function of the agent is as follows:

$$U_A(w,e) = \sqrt{w} - (e-1)$$

where

- w – wage rate,
- e is the effort level of the agent (1 or 2).

$$U'_A(w) = \frac{1}{2\sqrt{w}} > 0$$

$$U''_A(w) = \frac{1}{4\sqrt{w^3}} < 0$$

Copartnership

e \ R	10	30
1	p=2/3	p=1/3
2	p=1/3	p=2/3

$$ER_{e=1} = \frac{2}{3}10 + \frac{1}{3}30 = \frac{50}{3}$$

$$ER_{e=2} = \frac{1}{3}10 + \frac{2}{3}30 = \frac{70}{3}$$

Symmetrical information about efforts

$$EU_A[e \mid e=1] = \sqrt{w} - (e_1 - 1) = \sqrt{w} - (1-1) = 1 \Rightarrow w_1 = 1;$$

$$EU_A[e \mid e=2] = \sqrt{w} - (e_2 - 1) = \sqrt{w} - (2-1) = 1 \Rightarrow w_2 = 4;$$

$$EU_p[e \mid e=1] = ER_{e=1} - w_1 = \frac{2}{3}10 + \frac{1}{3}30 - 1 = \frac{47}{3}$$

$$EU_p[e \mid e=2] = ER_{e=2} - w_2 = \frac{1}{3}10 + \frac{2}{3}30 - 4 = \frac{58}{3}$$

- Therefore, the expected utility of the principal in the case of a high level of effort is greater, so that it will encourage the agent to choose a high level of effort.

Asymmetric information about the effort

- Restrictions on participation:

$$EU_p[e \mid e=2] = \sqrt{w} - (e_1 - 1) = \sqrt{w} - (1-1) = 1 \Rightarrow w = 1$$

- The contract has the form:

$$\begin{cases} w[R|R=10] = w_1 \\ w[R|R=30] = w_2 \end{cases}$$

- Alternative cost of labor of the agent:

$$EU_A[e \mid e=2] = \frac{1}{3}(\sqrt{w_1} - 1) + \frac{2}{3}(\sqrt{w_2} - 1) = 1$$

Expected utility of the agent

- The expected utility of the agent from a high level of effort should be equal to the expected utility when choosing a low level of effort, i.e.

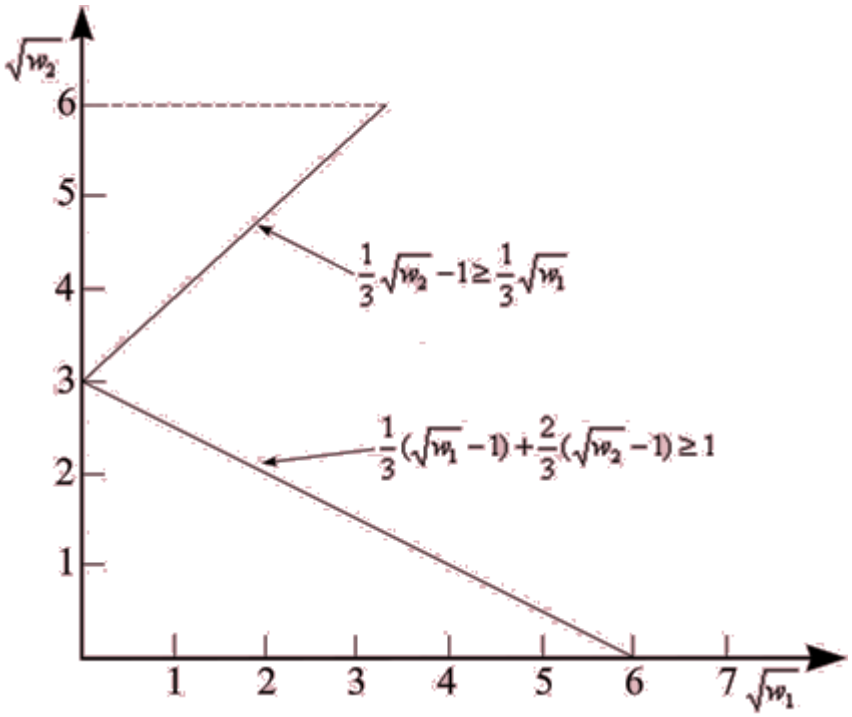
$$EU_p[e \mid e = 2] = EU_p[e \mid e = 1]$$

$$\frac{1}{3}(\sqrt{w_1} - 1) + \frac{2}{3}(\sqrt{w_2} - 1) = \frac{2}{3}(\sqrt{w_1} - 0) + \frac{1}{3}(\sqrt{w_2} - 0) \Rightarrow \frac{1}{3}\sqrt{w_2} - 1 = \frac{1}{3}\sqrt{w_1}$$

The problem of the principal

- The problem of the principal is to maximize its usefulness under these two limitations.
- The best terms of the contract provide

$w_1 = 0,\ w_2 = 9$



The optimal contract

$$EU_p[e \mid e=1] = \frac{2}{3}(10-0) + \frac{1}{3}(30-1) = \frac{47}{3}$$

$$EU_p[e \mid e=2] = \frac{1}{3}(10-0) + \frac{2}{3}(30-9) = \frac{52}{3}$$

- The principal must seek from the agent a high level of effort, so that the optimal contract looks like:

$$\begin{cases} w[R|R=10] = w_1 = 0 \\ w[R|R=30] = w_2 = 9 \end{cases}$$

The optimal contract in case of full bargaining power agent

- The agent's full bargaining power may be the result of competition between principals and agents, so that in order to attract agents, they set a reward for agents at which they themselves receive zero utility.
- In this case, the solution of the problem of finding the optimal contract is similar to the solution of the same problem for the case of full bargaining power of the principal. Only instead of the target function of the utility of the agent in the restriction of participation and self-selection is substituted by the utility function of the principal.

Symmetrical information about efforts

- First you need to find the amount of reward at two levels of effort that satisfy the limitations of participation:

$$EU_p[e \mid e=1] = \frac{2}{3}10 + \frac{1}{3}30 - w_1 = 0 \Rightarrow w_1 = \frac{50}{3}$$

$$EU_p[e \mid e=2] = \frac{1}{3}10 + \frac{2}{3}30 - w_2 = 0 \Rightarrow w_2 = \frac{70}{3}$$

- Substituting the amount of reward that would be offered by the principal in exchange, respectively, for low and high levels of effort, in the utility function of the agent, we obtain the value of utility:

$$EU_A[e \mid e=1] = \sqrt{w_1} - (e_1 - 1) = \sqrt{\frac{50}{3}} - (1-1) = 4,08$$

$$EU_A[e \mid e=2] = \sqrt{w_2} - (e_2 - 1) = \sqrt{\frac{70}{3}} - (2-1) = 3,83$$

- A contract with a low level of effort is optimal, the agent receives a reward of 50/3.

Asymmetric information about the effort

- Find the reward at two levels of effort, providing the principal zero profit. For the agent with a low level of effort, as in the case of symmetric information, it is sufficient to comply with the participation restriction:

$$EU_p[e \mid e=1] = \frac{2}{3}10 + \frac{1}{3}30 - w_1 = 0 \Rightarrow w_1 = \frac{50}{3}$$

Choice of agent

- The choice of a high level of effort by the agent requires the establishment of a relationship between the agent's remuneration and the principal's income:

$$EU_p[e \mid e=2] = \frac{1}{3}(10 - w_1) + \frac{2}{3}(30 - w_2) = 0 \Rightarrow w_1 = 70 - 2w_2;$$

$$EU_p[e \mid e=2] = EU_p[e \mid e=1] \Leftrightarrow$$

$$\frac{1}{3}(10 - w_1) + \frac{2}{3}(30 - w_2) = \frac{2}{3}10 + \frac{1}{3}30 - w_1$$

$$\Rightarrow w_1 = w_2 - 10.$$

The optimal contract

- Solving this system of equations gives two levels of reward $w_1 = 50/3$, $w_2 = 80/3$, and the corresponding contract will look like:

$$\begin{cases} w[R|R=10]=w_1=\dfrac{50}{3} \\ w[R|R=30]=w_2=\dfrac{80}{3} \end{cases}$$

Agent choice

- What level of effort in this case provides greater utility to the agent?

$$EU_A[e \mid e=1] = \sqrt{w_1} - (e_1 - 1) = \sqrt{\frac{50}{3}} - (1-1) = 4,08$$

$$EU_A[e \mid e=2] = \frac{1}{3}(\sqrt{w_1} - (e_2 - 1)) + (\sqrt{w_2} - (e_2 - 1)) =$$

$$= \frac{1}{3}(\sqrt{\frac{50}{3}} - 1) + \frac{2}{3}(\sqrt{\frac{80}{3}} - 1) = 3,81$$

The conclusion of the model

- A comparison of the utility values of the agent indicates that in the case of asymmetric information, as well as in the case of symmetric information, it is more profitable to choose a low level of effort, and the optimal contract will be similar to the optimal contract for symmetric information 50/3.

Thank you!